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Summary

# Virtuality Distributions and Pion Transition Form Factor

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# Transverse Momentum Distributions

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- Longitudinal Momentum Distribution

$$p_+^N \int_0^1 f(x)\, x^N \, dx = \langle p|\phi(0)\partial_+^N \phi(0)|p\rangle$$

- Transverse Momentum Distribution

$$\langle p|\phi(0)\partial_+^N (\partial_\perp^2)^l \phi(0)|p\rangle = p_+^N \int_0^1 f(x, k_\perp)\, x^N (k_\perp^2)^l \, dx$$

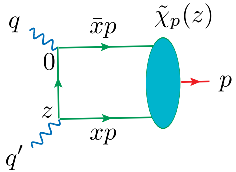
- Operators with $(\partial_\perp^2)^l$ : higher twists
- Usual twist decomposition of $\phi(0)\partial^{\mu_1} \ldots \partial^{\mu_n} \phi(0)$ involves matrix elements

$$\langle p|\phi(0)\{\partial^{\mu_1} \ldots \partial^{\mu_{n-2l}}\}(\partial^2)^l \phi(0)|p\rangle$$

  $[\{\ldots\} \equiv$ traceless] containing $\partial^2$ rather than $\partial_\perp^2$
- $\partial^2$ is related to parton virtuality
- Relate virtuality distributions with TMDs

# $\gamma^*\gamma \to \pi^0$ transition amplitude at twist 2






$$\gamma(q')\gamma^*(q) \to \pi^0(p)$$

$$q'^2 = 0\,,\ q^2 = -Q^2$$

- Twist-2 distribution amplitude:

$$\langle p|\phi(0)\phi(z)|0\rangle = \int_0^1 \varphi(x)\ e^{ix(pz)}\, dx + \mathcal{O}(z^2)$$

- Twist-2 transition amplitude (for $p^2 = 0$ and $(q' - p)^2 = -Q^2$)

$$T(p,q) = \int_0^1 dx \int d^4z\, e^{-i(q'z)+ix(pz)}\ D^c(z)$$

$$= \int_0^1 \frac{\varphi(x)}{-(q' - xp)^2}\, dx = \int_0^1 \frac{\varphi(x)}{xQ^2}\, dx$$

# Twist decomposition of bilocal operator



- Taylor expansion in bilocal operator $\phi(0)\phi(z)$

$$\phi(z) = \sum_{n=0}^{\infty} \frac{1}{n!} z_{\mu_1} \dots z_{\mu_n} \partial^{\mu_1} \dots \partial^{\mu_n} \phi(0)$$

- Twist expansion (with $\{z\partial\}^n \equiv \{z_{\mu_1} \dots z_{\mu_n}\} \partial^{\mu_1} \dots \partial^{\mu_n}$ )

$$\phi(z) = \sum_{l=0}^{\infty} \left(\frac{z^2}{4}\right)^l \sum_{N=0}^{\infty} \frac{N+1}{l!(N+l+1)!} \{z\partial\}^N (\partial^2)^l \phi(0)$$

- Virtuality-dependent matrix elements

$$\langle p|\phi(0)\{z\partial\}^k \left(\partial^2\right)^l \phi(0)|0\rangle \equiv [i\{zp\}]^k \Lambda^{2l} A_{kl}$$

- At handbag level, traceless combinations only complicate life. Simpler:

$$\langle p|\phi(0)\phi(z)|0\rangle = \sum_{l=0}^{\infty} \frac{1}{l!} \left(\frac{z^2}{4}\right)^l \sum_{N=0}^{\infty} i^N \frac{(pz)^N}{N!} B_N^{(l)} .$$

- Treat $B_N^{(l)}$ as double moments of virtuality distribution amplitude (VDA) $\Phi(x,\sigma)$

$$B_N^{(l)} = (-i)^l \int_0^\infty d\sigma\, \sigma^l e^{-\epsilon\sigma/4} \int_0^1 dx\, x^N\, \Phi(x,\sigma)$$

# Virtuality distributions



- VDA representation is double Fourier transform for a function of $(pz)$ and $z^2$

$$\langle p|\phi(0)\phi(z)|0\rangle = \int_0^\infty d\sigma \int_0^1 dx\, \Phi(x,\sigma)\; e^{ix(pz)-i\sigma(z^2-i\epsilon)/4}$$

- Less trivial: support properties $0 \le x \le 1$ and $\sigma \ge 0$
- Schwinger alpha-representation for any contributing diagram

$$\langle p|\phi(0)\phi(z)|0\rangle = \text{const} \int_0^\infty \prod_{j=1}^{l} d\alpha_j [A(\alpha)+B(\alpha)]^{-d/2}$$
$$\times \exp\left\{-i\frac{z^2/4}{A(\alpha)+B(\alpha)} + i(pz)\frac{B(\alpha)}{A(\alpha)+B(\alpha)}\right\}$$
$$\times \exp\left\{ip^2 C(\alpha) - i\sum_j \alpha_j (m_j^2 - i\epsilon)\right\}$$

with positive $A(\alpha)$, $B(\alpha)$, $C(\alpha)$
$\Rightarrow \sigma \equiv 1/(A+B) \ge 0$, $0 \le x \equiv B/(A+B) \le 1$

- No assumptions about finiteness of matrix elements $\langle p|\phi(0)\phi(z)|0\rangle$ for $z^2 = 0$ are necessary!
- Note: dependence on $z^2 - i\epsilon$, and $p$ is actual pion momentum, $p^2 = m_\pi^2$

# Transverse momentum dependent DAs



- Pion momentum is defined to have no transverse components
- Projection on $z^+ = 0$ interval $z = (z^-, z_\perp)$

$$\langle p|\phi(0)\phi(z)|0\rangle|_{z^+=0, p_\perp=0} = \int_0^1 dx\, d^2k_\perp\, \Psi(x, k_\perp)\, e^{i(k_\perp z_\perp)}\; e^{ix(pz^-)}$$

- TMDA can be written in terms of VDA (valid "always")

$$\Psi(x, k_\perp) = \frac{i}{\pi}\int_0^\infty \frac{d\sigma}{\sigma}\,\Phi(x,\sigma)\; e^{-i(k_\perp^2 - i\epsilon)/\sigma}$$

- Relation for moments (valid for soft functions)

$$\int \Psi(x, k_\perp)\, k_\perp^{2n}\, d^2k_\perp = \frac{n!}{i^n}\int_0^\infty \sigma^n\,\Phi(x,\sigma)\, d\sigma$$

- The question is whether TMDA appears in calculation of handbag diagram that involves $d^4z$ integral?

# Handbag diagram in VDA representation

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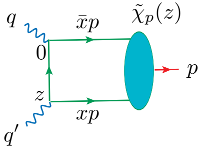



- Starting expression

$$T(p,q) = \int_0^1 dx \int d^4z \, e^{-i(q'z)+ix(pz)} \, D^c(z) \, \langle p|\phi(0)\phi(z)|0\rangle$$

- Using VDA representation

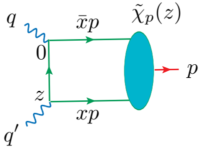
$$T(Q^2) = \int_0^1 \frac{dx}{xQ^2} \int_0^\infty d\sigma \, \Phi(x,\sigma) \left\{ 1 - e^{-[ixQ^2+\epsilon]/\sigma} \right\}$$

- First term: twist-2 approximation
- Integral of VDA over $\sigma$ may be written as integral of TMDA over $k_\perp$:

$$T(Q^2) = \int_0^1 \frac{dx}{xQ^2} \int_{k_\perp^2 \le xQ^2} \Psi(x,k_\perp) \, d^2k_\perp$$

# "Invisible" terms






- Massless scalar propagator is $\sim 1/z^2 \Rightarrow (z^2)^l$ terms from the $\langle p|\phi(0)\phi(z)|0\rangle$ expansion kill the singularity when $l \geq 1$
- Each of resulting $\Box_{q'}^{l-1}\delta^4(q'-xp)$ contributions is "invisible"
- But their sum gives a nontrivial function

$$-\int_{k_\perp^2 \geq xQ^2} \Psi(x,k_\perp)\, d^2k_\perp$$

- For a soft distribution $\Psi(x,k_\perp)$ it decreases for large $Q^2$ faster than any power of $1/Q^2$
- VDA result has strict cut-off $k_\perp^2 \leq xQ^2$. NOTE: popular cut-offs like

$$\int_0^1 \frac{dx}{xQ^2} \to \int dx\, d^2k_\perp \frac{\Psi(x,k_\perp)}{xQ^2+k_\perp^2}$$

generate infinite tower of $\sim 1/Q^{2l}$ corrections $\Rightarrow$ incompatible with OPE

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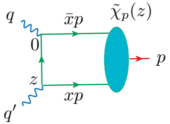


- Handbag contribution

$$\int d^4 z\, e^{-i(qz)} \langle p|\bar{\psi}(0)\gamma^\nu\, S^c(-z)\, \gamma^\mu\, \psi(z)|0\rangle = i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta F(Q^2)$$

- Antisymmetric part of $\gamma^\nu \not{z}\, \gamma^\mu$ is $iz_\beta \epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\alpha$, and we need

$$\langle p|\bar{\psi}(0)\,\gamma_5\gamma_\alpha\,\psi(z)|0\rangle = ip_\alpha \int_0^\infty d\sigma \int_0^1 dx\, \Phi(x,\sigma)\; e^{ix(pz)-i\sigma(z^2-i\epsilon)/4}$$

- Result in terms of VDA [based on $S^c(-z) \sim \not{z}/(z^2)^2$]

$$F(Q^2) = \int_0^\infty d\sigma \int_0^1 \Phi(x,\sigma)\frac{dx}{xQ^2}\left\{1 + \frac{i\sigma}{xQ^2}\left[1 - e^{-[ixQ^2+\epsilon]/\sigma}\right]\right\}$$

# Spin-1/2 quarks (cont.)



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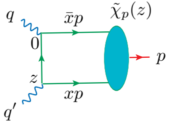


- Result in terms of TMDA

$$F(Q^2) = \pi \int_0^1 \frac{dx}{xQ^2} \int_0^{xQ^2} dk_\perp^2 \left[1 - \frac{k_\perp^2}{xQ^2}\right] \Psi(x, k_\perp)$$

- Finite for $Q^2 \to 0$:

$$F(Q^2 \to 0) = \frac{\pi}{2} \int_0^1 \Psi(x, k_\perp = 0)\, dx$$

- Result is natural in view of $k_\perp^2 \leq xQ^2$

# Gauge theories


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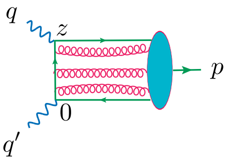



- In a covariant gauge handbag should be complemented by $\bar{\psi}(0) \ldots \not{A}(z_i) \ldots \psi(z)$ insertions of twist-0 gluonic field $A_{\mu_i}(z_i)$
- Can be organized into path-ordered exponential of zero-twist field $A^\mu$

$$E(0, z; A) \equiv P \exp\left[ig\, z_\mu \int_0^1 dt\, A^\mu(tz)\right]$$

- and insertions of non-zero twist gluon field

$$\mathfrak{A}^\mu(z) = z_\nu \int_0^1 G^{\mu\nu}(sz)\, s\, ds\ ,$$

- which is the vector potential in the Fock-Schwinger gauge

# VDA representation in gauge theories

- Two-body $\bar{q}q$ Fock component is given by gauge-invariant bilocal operator

$$\mathcal{O}^{\alpha}(0,z;A) \equiv \bar{\psi}(0)\,\gamma_5\gamma^{\alpha}\,E(0,z;A)\psi(z)$$

- Taylor expansion involves covariant derivatives $D^{\mu} = \partial^{\mu} - igA^{\mu}$

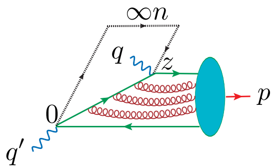
$$E(0,z;A)\,\psi(z) = \sum_{n=0}^{\infty} \frac{1}{n!}\, z_{\mu_1}\ldots z_{\mu_n}\; D^{\mu_1}\ldots D^{\mu_n}\psi(0)$$

- We can introduce VDA parametrization

$$\langle p|\,\mathcal{O}^{\alpha}(0,z;A)|0\rangle = ip^{\alpha}\int_0^{\infty} d\sigma \int_0^1 dx\,\Phi(x,\sigma)\; e^{ix(pz)-i\sigma(z^2-i\epsilon)/4}$$

- and proceed as in non-gauge case

# “Stapled” gauge links




- Another gauge-invariant bilocal operator

$$\mathcal{O}^{\alpha}_{n}(0,z;A) \equiv \bar{\psi}(0)E^{\dagger}_{n}(0;A)\,\gamma_5\gamma^{\alpha}\,E_n(z;A)\psi(z)$$

involves two infinite-length gauge links

$$E_n(z;A) \equiv P\exp\left[ig\int_0^{\infty} dt\, n_{\mu}\, A^{\mu}(z+tn)\, e^{-\epsilon t}\right]$$

- Taylor expansion involves “longer” covariant derivatives $\mathcal{D}^{\mu} = D^{\mu} + ig\mathfrak{A}^{\mu}_{n}$ with $\mathfrak{A}^{\mu}_{n}$ satisfying the axial gauge condition $n_{\mu}\mathfrak{A}^{\mu}_{n}(z) = 0$ and given by

$$\mathfrak{A}^{\mu}_{n}(z) = n_{\nu}\int_0^{\infty} dt\, G^{\mu\nu}(z+tn)\, e^{-\epsilon t}$$

- “Stapled” links are natural for pion EM form factor (process $\gamma^{*}\pi(p_1) \to \pi(p_2)$) with $n = p_2$ for $\langle 0|\ldots|p_1\rangle$ and $n = p_1$ for $\langle p_2|\ldots|0\rangle$

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- Generic VDA representation treats $(pz)$ and $z^2$ as independent variables

$$\langle p|\phi(0)\phi(z)|0\rangle \equiv F((pz), z^2) = \int_0^\infty d\sigma \int_0^1 dx\, \Phi(x,\sigma)\; e^{ix(pz)-i\sigma(z^2-i\epsilon)/4}$$

- Lorentz invariance is fully incorporated already
  $\Rightarrow$ no *a priori* correlation of $x$ and $\sigma$ dependence in VDA is demanded
- Simplest example: factorized models for VDA

$$\Phi(x,\sigma) = \varphi(x)\,\Phi(\sigma)$$

- Factorized models for TMDA

$$\Psi(x, k_\perp) = \varphi(x)\,\psi(k_\perp^2)/\pi$$

# Modeling soft TMDAs

- Gaussian dependence on $k_\perp$

$$\Psi_G(x,k_\perp) = \frac{\varphi(x)}{\pi\Lambda^2} e^{-k_\perp^2/\Lambda^2}$$

- Impact parameter Gaussian DA

$$\varphi_G(x,z_\perp) = \varphi(x)\, e^{-z_\perp^2 \Lambda^2/4}$$

- Faster fall-off at large $z_\perp$ compared to $\sim e^{-|z_\perp|m}$ of massive propagator

$$D^c(z,m) = \frac{1}{16\pi^2}\int_0^\infty e^{-i\sigma z^2/4 - i(m^2-i\epsilon)/\sigma} d\sigma$$

- But we need $\langle p|\phi(0)\phi(z)|0\rangle$ finite at $z^2=0$
- Add a constant term $(-4/\Lambda^2)$ to $z^2$ in the VDA representation, i.e. take

$$\Phi_m(x,\sigma;\Lambda) = \varphi(x)\frac{e^{i\sigma/\Lambda^2 - im^2/\sigma - \epsilon\sigma}}{2im\Lambda K_1(2m/\Lambda)} \; ; \; \Psi_m(x,k_\perp) = \varphi(x)\frac{K_0\left(2\sqrt{k_\perp^2+m^2}/\Lambda\right)}{\pi m\Lambda K_1(2m/\Lambda)}$$

- Concentrating on finite-size effects: take $m=0$ model

$$\Phi_{m=0}(x,\sigma;\Lambda) = \varphi(x)\frac{e^{i\sigma/\Lambda^2-\epsilon\sigma}}{i\Lambda^2} \; ; \; \varphi_{m=0}(x,z_\perp) = \frac{\varphi(x)}{1+z_\perp^2\Lambda^2/4}$$

# Modeling transition form factor

- Gaussian model

$$F_G(Q^2) = \int_0^1 \frac{dx}{xQ^2}\,\varphi(x)\,\left[1 - \frac{\Lambda^2}{xQ^2}\left(1 - e^{-xQ^2/\Lambda^2}\right)\right]$$

- Power-like (under $x$-integral) twist-4 contribution
- Formal $Q^2 \to 0$ limit is finite:

$$F_G(Q^2 = 0) = \frac{f_\pi}{2\Lambda^2} \;\; ; \;\; f_\pi \equiv \int_0^1 \varphi(x)\,dx$$

- Note: $F(Q^2)$ is finite for $Q^2 = 0$ in any model with finite $\Psi(x, k_\perp = 0)$

$$F(Q^2 = 0) = \frac{\pi}{2}\int_0^1 \Psi(x, k_\perp = 0)\,dx$$

- Non-Gaussian $m = 0$ model

$$F(Q^2) = \int_0^1 \frac{dx}{xQ^2}\,\varphi(x)\,\left[1 - \frac{\Lambda^2}{xQ^2} + 2K_2(2\sqrt{x}Q/\Lambda)\right]$$

- Size of twist-4 term is governed by confinement scale $\Lambda$

# Comparison with data



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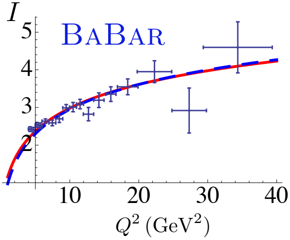
- In leading-order perturbative QCD

$$F^{\rm LOpQCD}(Q^2) = \int_0^1 \frac{dx}{xQ^2}\,\varphi(x) \equiv I(Q^2) f_\pi / Q^2$$

- For DAs $\varphi_r(x) \sim (x\bar{x})^r$, one has $I_r^{\rm LOpQCD}(Q^2) = 1 + 2/r$
- $I^{\rm as}(Q^2) = 3$ for "asymptotic" wave function $\varphi^{\rm as}(x) = 6 f_\pi x\bar{x}$
- Recent experimental data from BaBar and Belle do not show flattening yet




- Curves for BaBar data with flat DA $\varphi(x) = f_\pi$ and $\Lambda_G^2 = 0.35\,$GeV$^2$ or $\Lambda_{m=0}^2 = 0.6\,$GeV$^2$

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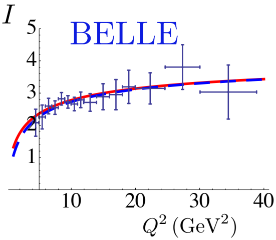


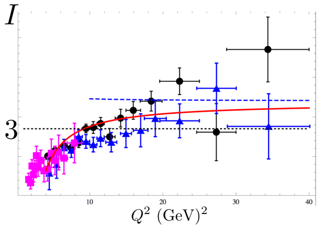


- Left: Curves for Belle data with $\varphi(x) \sim f_\pi (x\bar{x})^{0.4}$ and $\Lambda_G^2 = 0.3\,\text{GeV}^2$ or $\Lambda_{m=0}^2 = 0.4\,\text{GeV}^2$
- Right: Curve based on CZ DA $\varphi(x) \sim f_\pi (x\bar{x})(1-2x)^2$ with two ad hoc power corrections [Chernyak 2009] (BaBar – black, Belle – blue)

$$I^{CZ}(Q^2) = 3\left[1.19 - \frac{1.5\,\text{GeV}^2}{Q^2} - \left(\frac{1.2\,\text{GeV}^2}{Q^2}\right)^2\right]$$

# Modeling hard tail

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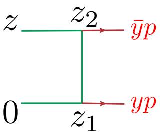
- Hard exchange model (Yukawa gluon)




$$\Phi^{\rm exch}(x,\sigma;y) = g^2\,\frac{e^{i(x\bar{x}p^2 - m^2)/\sigma}}{16\pi^2\,\sigma} \times \int_0^{\min\left\{\frac{x}{y},\,\frac{\bar{x}}{\bar{y}}\right\}} e^{-iy\bar{y}\beta\,p^2/\sigma}\,d\beta$$

- For $p^2 = 0$, $\beta$-integral gives part of ERBL evolution kernel

$$V(x,y) = \frac{x}{y}\,\theta(x<y) + \frac{\bar{x}}{\bar{y}}\,\theta(x>y)$$

- TMDA generated in $p^2 = 0$ limit (using $\alpha_g \equiv g^2/16\pi^2$)

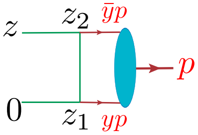
$$\Psi^{\rm exch}(x,k_\perp;y) = \frac{\alpha_g}{\pi}\,\frac{V(x,y)}{k_\perp^2 + m^2}$$

# Convolution model

- Superpose $yp, \bar{y}p$ states with the weight $\varphi_0(y)$ = "primordial" DA
- TMDA is given by a convolution

$$\Psi^{\mathrm{conv}}(x, k_\perp) = \frac{\alpha_g}{\pi} \frac{1}{k_\perp^2 + m^2} \int_0^1 V(x,y)\, \varphi_0(y)\, dy$$

- Use "primordial" soft TMDA $\Psi_0(y, k_\perp) \equiv \psi_0(x, k_\perp^2)/\pi$ (and $m = 0$)




$$\Psi^{B_0}(x, k_\perp) = \frac{\alpha_g}{\pi} \int_0^1 dy$$
$$\times \left[ \int_0^1 d\xi\, \psi_0 \left( y, \frac{\xi k_\perp^2}{V(x,y)} \right) \right]$$

- Term in square brackets may be written as

$$\left[ \cdots \right] = \frac{V(x,y)}{k_\perp^2} \int_0^{k_\perp^2/V(x,y)} \psi_0(y, {k'_\perp}^2)\, d{k'_\perp}^2$$

# Convolution model in momentum space



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- For large $k_\perp$, leading $1/k_\perp^2$ term is determined by DA $\varphi_0(y)$ only

$$\Psi^{B_0}(x,k_\perp) = \frac{\alpha_g}{\pi}\int_0^1 dy \frac{V(x,y)}{k_\perp^2}\left\{\varphi_0(y) - \int_{k_\perp^2/V(x,y)}^{\infty} \psi_0(y,{k'_\perp}^2)\, d{k'_\perp}^2\right\}$$

- $k_\perp \to 0$ limit is finite

$$\Psi_Y^{B_0}(x,k_\perp=0) = \alpha_g \int_0^1 dy\, \Psi_0\,(y,k_\perp=0)$$

- Using Gaussian model for $B_0$

$$\Psi_Y^{B_0,G}(x,k_\perp=0) = \alpha_g \frac{f_\pi}{\Lambda^2}$$

# Evolution in impact parameter space


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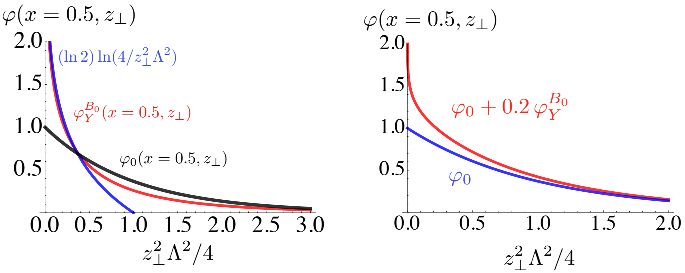
- In impact parameter space:

$$\varphi_Y^{B_0}(x, z_\perp^2) = \alpha_g \int_0^1 dy\, V(x,y) \int_1^\infty \frac{d\nu}{\nu}\, \varphi_0\Big(y, \nu\, z_\perp^2\, V(x,y)\Big)$$

- Integral over $\nu$ is cut at $\nu \sim 4/z_\perp^2\Lambda^2 \Rightarrow \ln(z_\perp^2\Lambda^2/4)$
- We can keep hard quarks massless
- Illustration for Gaussian model with flat DA $\varphi_0(x, z_\perp) = \exp[-z_\perp^2\Lambda^2/4]$

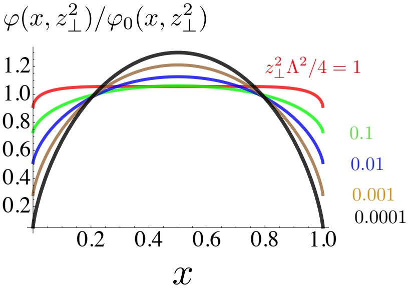


# Adding UV divergent correction

- Adding self-energy part (with $\mu = \Lambda/2$ and Bessel form for log singularity)

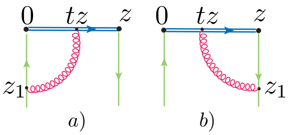
$$\delta\varphi_Y(x, z_\perp^2) = \alpha_g \left[ \int_0^1 dy\, V(x,y) \int_1^\infty \frac{d\nu}{\nu}\, \varphi_0\big(y, \nu\, z_\perp^2\, V(x,y)\big) - K_0(z_\perp \Lambda/2)\, \varphi_0(x, z_\perp^2) \right]$$

- Total IDA $\varphi(x, z_\perp^2) = \varphi_0(x, z_\perp^2) + \delta\varphi_Y(x, z_\perp^2)$
- Illustration for Gaussian model with flat DA $\varphi_0(x) = 1$ and $\alpha_g = 0.2$



# QCD case






- Need to add contribution due to gauge link

$$\psi_{\rm link}^{\rm h,total}(x,k_\perp^2) = \alpha_g \int_x^1 \frac{dy}{y-x} \int_0^1 \frac{d\xi}{\bar\xi} \times \left[ \psi_0\left(y, \frac{\xi k_\perp^2}{x/y}\right) - \psi_0\left(y, \frac{\xi^2 k_\perp^2}{x/y - \bar\xi}\right) \theta\left(\bar\xi \le \frac{x}{y}\right) \right] + \alpha_g \int_0^1 d\tau \, \frac{\ln \tau}{1-\tau} \, \psi_0\left(x, \tau k_\perp^2\right) + \left\{ y \to \bar y \, , \; x \to \bar x \right\} .$$

- $\alpha_g = \alpha_s C_F/\pi$
- Evolution pattern is essentially the same

# Summary



- A new approach to transverse momentum dependence that starts with a covariant calculation
- New object: virtuality distribution $\Phi(x, \sigma)$
- Introduced transverse momentum distribution $\Psi(x, k_\perp)$ and wrote it in terms of $\Phi(x, \sigma)$
- Technically: Exact integration over $k^-$ momentum in 4-dimensional handbag integral for $\gamma\gamma^* \to \pi^0$
- The result given in terms of light-front WF/TMDA, and contains contributions invisible in OPE
- Proposed simple models for soft VDAs/TMDAs, and used them for comparison with experimental data on the pion transition form factor
- Nontrivial shape of $Q^2 F(Q^2)$ observed up to $Q^2 \sim 40 \mathrm{GeV}^2$ is dominated by “invisible” terms
- Hard tails are generated from soft primordial TMDAs $\Rightarrow$ natural interpolation between small- and large-$k_\perp^2$ regions

# Future directions



Twist decomposition
VDA
TMDA
Handbag in VDA
Spin-1/2 quarks
Gauge theories
Modeling TMDAs
Modeling form factor
Modeling hard tail
Hard exchange
Convolution model
Summary

- Extension of VDF approach onto inclusive processes, work in progress: inclusive DIS
- Building VDF-based models for soft parts of TMDs that would have a non-Gaussian behavior at large $k_\perp$
- Generating hard tails from these soft TMDs, thus solving the problem of interpolation between small- and large-$k_\perp^2$ regions
- Extension onto GPDs,....